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STRESS-STRAIN STATES IN A MULTISHEET SURFACE WITH CUTS*

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The first, second and mixed fundamental boundary-value problems of elasticity theory are considered on an n-sheet Riemann surface with straight-line cuts joining the branch points. The cuts are such that their edges are situated in different planes. Complex potentials are constructed, asymptotic representations of the stresses and derivatives of the displacement components are obtained near the vertices of the cuts and invariant Γ- integrals /1/ are obtained, by the method of reduction to a matrix Riemann boundary-value problem.

The first and second fundamental problems for an $n=2$ Riemann surface were solved /2/ by the Riemann boundary-value problem method for a Riemann surface. For $n=1$ the results are identical with previously known results for a plane /3/.

1. Statement of the problem. Suppose we have n identical thin homogeneous isotropic elastic infinite plates $E_1, E_2, \ldots, E_n$ of the same thickness and with cuts along the same intervals $l_j=[a_j, b_j]$ $(j=1, 2, \ldots, m)$ along the real x axis superimposed on one another so that, for all the plates, cuts with the same numbers are placed above one other. The lower edges of the plate E_k are glued to the corresponding upper edges of plate E_{k+1} $(k=1, 2, \ldots, n-1)$. The upper edges of the cuts of E_1 and the lower edges of E_n are not glued together. We shall denote them by L^+ and L^- respectively. If one takes a section perpendicular to

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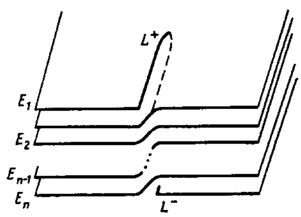

the plates E_k and perpendicularly intersecting one of the cuts l_j, then the resulting system will appear as in the figure. It is an n-sheeted Riemann surface for the algebraic function

$$w = \left(\prod_{j=1}^{m} \frac{z-a_j}{z-b_j}\right)^{1/n}, \quad z = x + iy \tag{1.1}$$

with boundary $L^+ \bigcup L^-$. If the edges L^+ and L^- are imagined as being glued together, i.e. identical, then we would obtain a closed Riemann surface R for the function (1.1). On this surface L^+ and L^- are the edges of three-dimensional cuts with ends at the branch points $z = a_j$ and $z = b_j$ of the surface.

Suppose all the sheets of the surface are in a generalized plane stressed state, characterized by the following conditions.

1. The stresses and displacements change continuously across the gluing lines of the sheets, while on the unglued edges L^+ and L^- either the normal and shear stresses σ_y^+, τ_{xy}^+ and σ_y^-, τ_{xy}^- (the first fundamental problem on the surface R) are specified, or the partial derivtives with respect to x of the displacement components $(u', v')^+$ and $(u', v')^-$ are specified (the second fundamental problem), or σ_y^+, τ_{xy}^+ are specified on L^+ and $(u', v')^-$ are specified on L^- (mixed fundamental problem). The specified boundary conditions of the stresses and displacement component derivatives will be assumed to be H-continuous, and in the second problem

$$\int_{l_j} [(u' + iu')^+ - (u' + iv')^-]\, dx = 0, \quad j = 1, 2, \ldots, m \tag{1.2}$$

because the displacement increment along the upper edge of cut l_j on the plate E_1 is equal to the displacement increment along the lower edge of the same cut on the plate E_n.

2. At the ends of the intervals l_j, i.e. at the branch points of the surface, the stress and displacement derivatives can have infinities of order less than unity, while at the remaining points of the intervals they are continuous.

3. At ∞ on the plates E_k $(k = 1, 2, \ldots, n)$ the stresses are uniformly distributed and here the principal stresses $(\sigma_1)_k$ and $(\sigma_2)_k$ make angles φ_k and $\varphi_k + \pi/2$ respectively with the real axis. The rotation of the plate E_k at ∞ is equal to ω_k.

4. The three-dimensional effect of the stress concentration at the joints of the sheets is assumed to be negligibly small.

Under these conditions the stresses $(\sigma_x, \sigma_y, \tau_{xy})_k$ and the x-derivatives of the displacement components $(u', v')_k$ in plate E_k are expressed in terms of two functions $\Phi_k(z)$, $\Psi_k(z)$ by the formulae /3/

$$(\sigma_x + \sigma_y)_k = 4\operatorname{Re}\Phi_k(z) \tag{1.3}$$

$$(\sigma_y - i\tau_{xy})_k = \Phi_k(z) + \Omega_k(\bar z) + (z - \bar z)\overline{\Phi_k'(z)}$$

$$2\mu(u' + iv')_k = \varkappa\Phi_k(z) - \Omega_k(\bar z) - (z - \bar z)\overline{\Phi_k'(z)}$$

$$\Omega_k(z) = \bar\Phi_k(z) + z\bar\Phi_k'(z) + \bar\Psi_k(z) \tag{1.4}$$

where μ is the shear modulus, $\varkappa = (3 - \nu)/(1 + \nu)$, and ν is Poisson's ratio, which are the same for all the plates. The functions Φ_k, Ω_k are analytic and single-valued in the E_k plane with cuts l_j $(j = 1, 2, \ldots, m)$, and in the neighbourhood of ∞ have the form

$$\Phi_k(z) = \gamma_k - \frac{P_k}{2\pi(1+\varkappa)}\frac{1}{z} + O(z^{-2}) \tag{1.5}$$

$$\Omega_k(z) = \gamma_k' + \frac{\varkappa P_k}{2\pi(11+\varkappa)}\frac{1}{z} + O(z^{-2})$$

$$\gamma_k = \frac{1}{4}(\sigma_1 + \sigma_2)_k + \frac{2i\mu}{1+\varkappa}\omega_k, \quad \gamma_k' = \bar\gamma_k + \frac{1}{2}(\sigma_2 - \sigma_1)_k \exp(2i\varphi_k)$$

where $-P_k = -(X_k + iY_k)$ is the total force applied to the collection of cuts l_j $(j = 1, 2, \ldots, m)$ from the side of plate E_k. We assume that the P_k $(k = 1, 2, \ldots, n)$ have been specified, and that for the first problem these numbers and the specified boundary conditions on the stress satisfy the equilibrium condition for the surface R:

$$\sum_{k=1}^{n} P_k + \sum_{j=1}^{m} \int_{l_j} [(\tau_{xy} + i\sigma_y)^+ - (\tau_{xy} + i\sigma_y)^-]\, dx = 0 \tag{1.6}$$

At the ends of the intervals l_j the functions Φ_k, Ω_k can have infinities of order less than unity, while at the other points of the intervals they have continuous boundary values. Furthermore, we suppose that at all points $t \in l_j$ except at the ends

$$(z - \bar{z})\,\Phi_k'(z) \to 0 \quad \text{for} \quad z \to t\pm \tag{1.7}$$

In the problems considered below this condition is satisfied as a consequence of the H-continuity of the specified boundary conditions.

Using (1.3), (1.7) the boundary conditions on the edges L^+, L^- can be written in a unified form for all the problems (L is the set of all intervals l_j, ($j = 1, 2, \ldots, m$)):

$$\rho_1\Phi_1^+(t) + \Omega_1^-(t) = f_1(t), \quad \rho_2\Phi_n^-(t) + \Omega_n^+(t) = f_2(t), \quad t \in L \tag{1.8}$$

Here, in the first problem,

$$\rho_1 = \rho_2 = 1, \quad f_1 = (\sigma_y - i\tau_{xy})^+, \quad f_2 = (\sigma_y - i\tau_{xy})^-$$

In the second problem

$$\rho_1 = \rho_2 = -\varkappa, \quad f_1 = -2\mu\,(u' + iv')^+, \quad f_2 = -2\mu\,(u' + iv')^-$$

In the mixed problem

$$\rho_1 = 1, \quad \rho_2 = -\varkappa, \quad f_1 = (\sigma_y - i\tau_{xy})^+, \quad f_2 = -2\mu\,(u' + iv')^-$$

the continuity of the stresses and displacements along the glue lines of the sheets is described by

$$\Phi_k^-(t) + \Omega_k^+(t) = \Phi_{k+1}^+(t) + \Omega_{k+1}^-(t)$$
$$\varkappa\Phi_k^-(t) - \Omega_k^+(t) = \varkappa\Phi_{k+1}^+(t) - \Omega_{k+1}^-(t), \quad t \in L,\ k = 1, 2, \ldots, n-1$$

whence

$$\Phi_k^-(t) = \Phi_{k+1}^+(t), \quad \Omega_k^+(t) = \Omega_{k+1}^-(t), \quad t \in L,\ k = 1, 2, \ldots, n-1 \tag{1.9}$$

Thus to find the functions Φ_k, Ω_k ($k = 1, 2, \ldots, n$) we have a Riemann matrix boundary-value problem (1.8), (1.9) which we write in the form

$$\Phi^+(t) = A\Phi^-(t) + f(t), \quad t \in L \tag{1.10}$$

where $\Phi(z)$ is an unknown piecewise-holomorphic vector function of order $2n$ with components $\Phi_1, \Phi_2, \ldots, \Phi_n, \Omega_1, \Omega_2, \ldots, \Omega_n$; $f(t)$ is a vector function of order $2n$ with components f_1/ρ_1, $0, 0, \ldots, 0, f_2$; $A = (A_{kj})$ is a matrix of order $2n \times 2n$ in which all the elements are zero except $A_{1,n+1} = -1/\rho_1$, $A_{k,k-1} = 1$, $k = 2, 3, \ldots, n$, $A_{k,k+1} = 1$, $k = n+1, n+2, \ldots, 2n-1$, $A_{2n,n} = -\rho_2$. The function $\Phi(z)$ can have infinities of order less than unity at the ends of the lines of L, while from (1.5) it has the form

$$\Phi(z) = G + Hz^{-1} + O(z^{-2}) \tag{1.11}$$

in a neighbourhood of ∞, G and $2\pi(1+\varkappa)H$ being $2n$-dimensional vectors with components $\gamma_1, \gamma_2, \ldots, \gamma_n, \gamma_1', \gamma_2', \ldots, \gamma_n'$ and $-P_1, -P_2, \ldots, -P_n, \varkappa P_1, \varkappa P_2, \ldots, \varkappa P_n$ respectively, and $O(z^{-2})$ is a vector function all of whose components are comparable with z^{-2} at large z.

2. ***Solution of the problem.*** We denote the eigenvalue of the matrix A by λ_k ($k = 1, 2, \ldots, 2n$), while S is a matrix whose columns are the eigenvectors of A. We straightforwardly find that for the first and second problems

$$\lambda_k = \exp[i\pi(k-1)/n], \quad k = 1, 2, \ldots, 2n \tag{2.1}$$

and for the mixed problem

$$\lambda_k = \varkappa^{1/2n}\exp[i\pi(2k-1)/(2n)]$$

while for $S = (S_{kj})$ we can take the matrix with elements

$$S_{kj} = \lambda_j^{1-k}, \quad k = 1, 2, \ldots, n, \quad S_{kj} = -\rho_1\lambda_j^{k-n}, \quad k = n+1, n+2, \ldots, 2n \tag{2.2}$$

Then /4/ the matrix $S^{-1}AS$ is diagonal with diagonal elements $\lambda_1, \lambda_2, \ldots, \lambda_{2n}$. Searching for $\Phi(z)$ in the form $\Phi(z) = SF(z)$ where $F(z)$ is a new unknown vector function with components $F_1, F_2, \ldots, F_{2n}$, we obtain from (1.10)

$$F_k^+(t) = \lambda_k F_k^-(t) + g_k(t), \quad t \in L,\ k = 1, 2, \ldots, 2n \tag{2.3}$$

where the g_k are the components of the vector function $S^{-1}f(t)$. At the ends of the lines of L the functions F_k can have infinities of order less than unity, while in the neighbourhood of infinity (1.11) gives

$$F(z) = (F_k) = S^{-1}\Phi = S^{-1}G + S^{-1}Hz^{-1} + O(z^{-2}) \tag{2.4}$$

According to /5/, the solutions of (2.3) are the functions

$$F_k(z) = X_k(z)\left(\frac{1}{2\pi i}\int_L \frac{g_k(t)}{X_k^+(t)}\frac{dt}{t-z} + \sum_{j=0}^{m} c_{kj}z^j\right) \tag{2.5}$$

$$X_k(z) = \prod_{j=1}^{m}\left(\frac{z-b_j}{z-a_j}\right)^{\alpha_k - i\beta}\frac{1}{z-b_j}, \quad k = 1, 2, \ldots, 2n$$

where for the first and second problems $\alpha_k = (k-1)/(2n)$ and $\beta = 0$, while for the mixed problem $\alpha_k = (2k-1)/(4n)$ and $\beta = (\ln \varkappa)/(4\pi n)$. Here $X_k(z)$ is to be understood as the branch that is single-valued in the plane with cuts along the lines L and such that $z^m X_k(z) \to 1$ as $z \to \infty$. In the first and second problems we must put

$$X_1(z) = 1, \quad c_{11} = c_{12} = \ldots = c_{1m} = 0 \tag{2.6}$$

because $\lambda_1 = 1$.

Using (2.6) in (2.4) and (2.5), we find that in the first and second problems the vector with components $c_{10}, c_{2m}, c_{3m}, \ldots, c_{2n,m}$ is equal to $S^{-1}G$, while the vector with components

$$-\left(\int_L g_1(t)\,dt\right)\Big/(2\pi i), \quad c_{k,m-1} + q_k c_{km}, \quad k = 2, 3, \ldots, 2n$$

$$(q_k = (\alpha_k - i\beta)(a_1 + a_2 + \ldots + a_m) + (1 - \alpha_k + i\beta)(b_1 + b_2 + \ldots + b_m))$$

is equal to $S^{-1}H$. The fact that the quantity $-\left(\int g_1 dt\right)/(2\pi i)$ is equal to the first component of the vector $S^{-1}H$ follows in the first problem from condition (1.6), and in the second problem from conditions (1.2). In the mixed problem we find from (2.4) and (2.5) that

$$(c_{km})_{k=1,2,\ldots,2n} = S^{-1}G, \quad (c_{k,m-1} + q_k c_{km})_{k=1,2,\ldots,2n} = S^{-1}H$$

where q_k is found from the same formulae as for the first and second problems.

Consequently, if the number of cuts $m = 1$ all the constants c_{kj} are defined. If $m > 1$, then to determine the remaining contants c_{kj} in the first problem one must require the increment of the displacements along a closed curve consisting of the edges of the cuts l_j $(j = 1, 2, \ldots, m-1)$ on each plate E_k $(k = 1, 2, \ldots, n)$ to vanish and the increment of the displacements along the interval $[b_j, a_{j+1}]$ $(j = 1, 2, \ldots, m-1)$ on the plate E_1 to be equal to the increment of the displacement along the same interval of the plate E_k $(k = 2, 3, \ldots, n)$. Then

$$\sum_{\nu=1}^{2n}(nS_{k\nu} + S_{k+n,\nu})\int_{l_j}[F_\nu^+(t) - F_\nu^-(t)]\,dt = 0, \quad k = 1, 2, \ldots, n \tag{2.7}$$
$$j = 1, 2, \ldots, m-1$$

$$\sum_{\nu=1}^{2n}[\varkappa(S_{1\nu} - S_{k\nu}) + S_{k+n,\nu} - S_{n+1,\nu}]\int_{b_j}^{a_{j+1}} F_\nu(t)\,dt = 0, \quad k = 2, 3, \ldots, n \tag{2.8}$$
$$j = 1, 2, \ldots, m-1$$

Substituting the values of F_k into these equations, we obtain a system of $(2n-1)(m-1)$ linear algebraic equations to determine the remaining $(2n-1)(m-1)$ constants c_{kj} $(k = 2, 3, \ldots, 2n;\ j = 0, 1, \ldots, m-2)$, unique solvability being proved by the usual methods /3/.

In the second problem one of the group of conditions (2.7), for example for $k = 1$, is a consequence of the remaining conditions and conditions (1.2), hence instead of them one should set an additional $m-1$ conditions. They can be obtained if the differences of the displacements of the points b_j and a_{j+1}. Then

$$\sum_{\nu=1}^{2n}(\varkappa S_{1\nu} - S_{n+1,\nu})\int_{b_j}^{a_{j+1}} F_\nu(t)\,dt = \tag{2.9}$$
$$2\mu[u(a_{j+1}) + iv(a_{j+1}) - u(b_j)\,iv(b_j)], \quad j = 1, 2, \ldots, m-1$$

Instead of the differences of the displacements of points b_j and a_{j+1} one can also specify the total external force vector acting on the l_j^+ side in plate E_1 or the l_j^- side in plate E_n, or on the combined sides l_j^+ in E_1 and l_j^- in E_n. Then for each j ($j = 1, 2, \ldots, m-1$) one of the following conditions should be satisfied:

$$\sum_{\nu=1}^{2n} \int_{l_j} [S_{1\nu} F_\nu^+(t) + S_{n+1,\nu} F_\nu^-(t)]\, dt = iQ_{j1} \tag{2.10}$$

$$\sum_{\nu=1}^{2n} \int_{l_j} [S_{n\nu} F_\nu^-(t) + S_{2n,\nu} F_\nu^+(t)]\, dt = -iQ_{jn} \tag{2.11}$$

$$\sum_{\nu=1}^{2n} \int_{l_j} [(S_{1\nu} - S_{2n,\nu}) F_\nu^+(t) + (S_{n+1,\nu} - S_{n\nu}) F_\nu^-(t)]\, dt = iQ_j \tag{2.12}$$

where Q_{j1}, Q_{jn} and $Q_j = Q_{j1} + Q_{jn}$ are the total vectors of external forces acting respectively on the l_j^+ side in E_1, on the l_j^- side in E_n, and on the combined sides l_j^+ in E_1 and l_j^- in E_n. Conditions (2.7), where $k = 2, 3, \ldots, n$, (2.8) and one of conditions (2.9)-(2.12) for each j form a uniquely solvable system of $(2n-1)(m-1)$ equations to determine the remaining $(2n-1)(m-1)$ unknown constants c_{kj}.

In the mixed problem, to determine the $2n(m-1)$ constants c_{kj} ($k = 1, 2, \ldots, 2n$; $j = 0, 1, \ldots, m-2$) one must take conditions (2.7), (2.8) and for each j one of conditions (2.9)-(2.12).

3. *Behaviour of the stress and displacement near the ends of the cuts. Invariant* Γ-*integrals.* For the first and second problems the functions $F_k(z)$ near the point $z = b_j$ have the form /5/

$$F_1(z) = O(\ln|z - b_j|) \tag{3.1}$$

$$F_k(z) = D_{kj}(z - b_j)^{\alpha_k - 1 - i\beta}, \quad k = 2, 3, \ldots, 2n \tag{3.2}$$

$$D_{kj} = \eta_{kj}(b_j)\Big(\frac{1}{2\pi i} \int_L \frac{g_k(t)}{X_k^+(t)} \frac{dt}{t - b_j} + \sum_{\nu=0}^{m} c_{k\nu} b_j^\nu\Big) \tag{3.3}$$

$$\eta_{kj}(z) = X_k(z)(z - b_j)^{1 - \alpha_k + i\beta}$$

where $\alpha_k = (k-1)/(2n)$, $\beta = 0$, the functions X_k, g_k and the numbers $c_{k\nu}$ are determined in Sect.2, while $(z - b_j)^{\alpha_k - 1 - i\beta}$ are single-valued branches in the plane with a cut along the ray $(-\infty, b_j]$ of the real axis, taking the value 1 at $z - b_j = 1$. The integral in (3.3) is improper.

In the mixed problem all the functions F_k, including F_1, have the form (3.2), where $\alpha_k = (2k-1)/(4n)$ and $\beta = (\ln \varkappa)/(4\pi n)$.

Because the vector function $\Phi(z)$ with components $\Phi_1, \Phi_2, \ldots, \Phi_n, \Omega_1, \Omega_2, \ldots, \Omega_n$ is equal to $SF(z)$, from (1.3), (3.1) and (3.2) we obtain for the first and second problems the following asymptotic representations of the stresses and displacement derivatives near the point $z = b_j$ in the plane E_k ($k = 1, 2, \ldots, n$):

$$(\sigma_x + \sigma_y)_k = 4\,\mathrm{Re}\Big(\sum_{\nu=2}^{2n} S_{k\nu} D_{\nu j} \omega_{\nu j}(z)\Big) + O(\ln r) \tag{3.4}$$

$$\begin{Bmatrix} (\sigma_y - i\tau_{xy})_k \\ 2\mu(u' + iv')_k \end{Bmatrix} = \begin{Bmatrix} 1 \\ \varkappa \end{Bmatrix} \Big(\sum_{\nu=2}^{2n} S_{k\nu} D_{\nu j} \omega_{\nu j}(z)\Big) + \begin{Bmatrix} 1 \\ -1 \end{Bmatrix} \Big(\sum_{\nu=2}^{2n} [S_{k+n,\nu} D_{\nu j} \omega_{\nu j}(\bar z) +$$

$$(\alpha_k + i\beta)\bar S_{k\nu} \bar D_{\nu j}(1 - (z - b_j)/\overline{(z - b_j)})\overline{\omega_{\nu j}(z)}]\Big) + O(\ln r)$$

$$r = |z - b_j|, \quad \omega_{\nu j}(z) = (z - b_j)^{\alpha_k - 1 - i\beta}, \quad \alpha_k = (k-1)/(2n), \beta = 0$$

The constants S_{kj} and $D_{\nu j}$ are given by formulae (2.1), (2.2), and (2.3), respectively, for the mixed problem all the sums over ν in these representations must be taken from 1 to $2n$, and we must put $\alpha_k = (2k-1)/(4n)$ and $\beta = (\ln \varkappa)/(4\pi n)$. In order to obtain representations near the point $z = a_j$ on the plate E_k, one must replace b_j with a_j and $\alpha_k - 1 - i\beta$ with $i\beta - \alpha_k$ in formulae (3.1)-(3.4).

For the $n = 1$ case of (3.4) we obtain previously known representations of stresses and displacement derivatives near the vertices of cracks and rigid sharp-angled inclusions /6/ by denoting the constant $2\sqrt{2}\rho_1 D_{2j}$ by $k_1 - ik_2$ (k_1 and k_2 being coefficients of the stress intensity).

For the first and second problems we compute the invariant Γ-integral of the first kind /1/ along a curve consisting of n identical circles Λ of small radius $r\,(r \ll 1)$ with centres at the points $z = b_j$, positioned on the plates $E_k\ (k = 1, 2, \ldots, n)$. Consistent with formula (2.7), from /7/ we have, using relation (1.4),

$$\Gamma = \Gamma_1 + i\Gamma_2 = \frac{i(\varkappa+1)}{2\mu} \sum_{k=1}^{n} \Big(\int_\Lambda \Omega_k(\bar z)\, \overline{\Phi_k(z)\, dz} - \operatorname{Re} \int_\Lambda \Phi_k^2(z)\, dz \Big)$$

Because

$$\Phi_k = \sum_{\nu=1}^{2n} S_{k\nu} F_\nu, \quad \Omega_k = \sum_{\nu=1}^{2n} S_{k+n,\,\nu} F_\nu$$

then using (3.1), (3.2) and integrating using polar coordinates given by $z - b_j = re^{i\theta}$, $-\pi \leqslant \theta \leqslant \pi$, we find

$$\Gamma_1 = -\frac{\pi n(\varkappa+1)\rho_1}{\mu} \operatorname{Re}\Big(\sum_{\nu=2}^{2n} e^{i\pi(\nu-1)/n} D_{\nu j} \bar D_{2n+2-\nu,\, j} \Big) \tag{3.5}$$

where $\rho_1 = 1$ in the first problem and $\rho_1 = -\varkappa$ in the second, while the $D_{\nu j}$ are defined by formulae (3.3). The Γ_2 component has the form

$$\Gamma_2 = \xi_0 + \xi_1 r^{-1/(2n)} + \xi_2 r^{-2/(2n)} + \ldots + \xi_{2n-2} r^{-(2n-2)/(2n)}$$

where the ξ_k are some generally non-zero constants.

For $n = 1$, denoting $2\sqrt{2\pi}\rho_1 D_{2j}$ by $k_1 - ik_2$, we obtain from (3.5) the well-known formula /1/

$$\Gamma_1 = \operatorname{Re}\Gamma = (\varkappa+1)(k_1^2 + k_2^2)/(8\mu\rho_1)$$

where in the first problem $\rho_1 = 1$ and in the second $\rho_1 = -\varkappa$.

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